Chapter Two

Binding Energy and Nuclear Models

(2-1)Binding Energy

Binding energy can be defined as the work needed to break the nucleus into its components (protons and neutrons) or it is the energy released when the components of the nucleus are combined together. Experiments have shown that the masses of the nucleus components are always greater than the mass of the nucleus, The reason for this is due to the nature of the composition of the nucleus, which contains protons and neutrons that are bound together within a small space (the nucleus). In order for these components to connect with each other, they need energy taken from the mass of the nucleus according to Einstein's equation ($E=mc^2$) (which expresses the equivalence between energy and mass), and because the mass of the nucleus depends mainly on the masses of its components, so the mass of the nucleus is less than the masses of its components, and it has been proven This is practically when measuring the mass of the nucleus by mass spectrometry, as the difference between the mass of the nucleus and the masses of its components is called Mass Defect) and can be represented by the following equation:

$$\Delta M = [Zm_p + Nm_N - M(A,Z)] \quad \textbf{...(2-1)}$$

m_p: The mass of a proton is equal to ($1.0072766amu$).

M_n: The mass of a neutron is equal to**(**$1.0086650amu$**).**

M(A,Z): nucleus mass.

When the lack of mass is converted into energy according to Einstein's equation, binding energy is produced, which can be represented by the following equation:

$$B.E = \Delta MC^2 \qquad \text{---------} \ (2\text{-}2)$$

$$B.E = [Zm_p + Nm_N - M(A,Z)]C^2 \qquad \text{---------} \ (2\text{-}3)$$

But in most tables, the mass of the atom is given instead of the mass of the nucleus, so equation (3-2) can be written as follows:

$$B.E = [Zm_H + Nm_N - M(A,Z)]C^2 \qquad \text{----------} \ (2\text{-}4)$$

m_H: The mass of a hydrogen atom is ($1.007825 amu$).

Note from equation (2-4) replacing the mass of the proton with the mass of the hydrogen atom because it contains an electron, in order to delete the electron masses in the atomic masses of the elements (see Appendix - II-).

The binding energy per nucleon can be defined as the ratio between the nuclear binding energy and the mass number of the nucleus, ie:

$$\xi = \frac{B.E}{A} \qquad ...(2\text{-}5)$$

(2-2) Other Formulas of Binding Energy

The bonding energy of the nucleus can be studied as being composed of other nuclei. For example, we consider the sulfur nucleus ($^{32}_{16}S_{16}$) to be composed of two oxygen nuclei ($^{16}_{8}O_8$) and for the purpose of calculating the bonding energy of sulfur in relation to the two oxygen nuclei that make up it, we follow the following:

$${}^{32}_{16}S_{16} \rightarrow {}^{16}_{8}O_{8} + {}^{16}_{8}O_{8}$$

$$B.E = [2M({}^{16}_{8}O_{8}) - M({}^{32}_{16}S_{16})]C^{2}$$

as:

$$1(amu) = 931.5 \ MeV / c^{2}$$

so the:

$$B.E = [2 \times 15.994915 - 31.972072] \times 931.5$$

$$B.E = [0.017758] \times 931.5$$

$$B.E = 16.54 MeV$$

To calculate the total bonding energy, all components of the nucleus (protons and neutrons) are used, as follows:

$$B.E = [ZM_{H} + NM_{N} - M(A, Z)]C^{2}$$

$$B.E = [16 \times 1.007825 + 16 \times 1.008665 - 31.972072] \times 931.5$$

$$B.E = [16.1252 + 16.13864 - 31.972072] \times 931.5$$

$$B.E = [32.26384 - 31.972072] \times 931.5$$

$$B.E = [0.291768] \times 931.5$$

$$B.E = 272 \ MeV$$

The bonding energy (MeV16.54) is much lower than the total bonding energy of the sulfur nucleus (MeV272) and this result is intuitive because the two oxygen nuclei have self-bonding energy, which represents the bonding energy (16) nucleons for each nucleus.

To calculate the emission energy of a helium nucleus (alpha particle) from some nuclei, especially the heavy and radioactive ones for these particles, we take the uranium nucleus (${}^{238}_{92}U_{146}$) as it is composed of a helium nucleus (${}^{4}_{2}He_{2}$) and a thorium nucleus (${}^{234}_{90}Th_{144}$), its bonding energy will be:

$$B.E({}^{238}_{92}U_{146}) = [\mathrm{M}({}^{234}_{90}Th_{144}) + m({}^{4}_{2}He_{2}) - M({}^{238}_{92}U_{146})]\ C^{2}$$

$$B.E({}^{238}_{92}U_{146}) = -4.28\ MeV$$

The negative value of the binnding energy means that the uranium nucleus decomposes into a helium nucleus and a thorium nucleus, and this is a certain fact, since uranium nuclei have radioactivity in which these nuclei radiate, and this energy can be called the emission energy of a helium nucleus and is symbolized by (ε_{α}), but when it is positive, it is known It is the energy needed to separate a helium nucleus from a uranium nucleus, where the general formula for it can be written as follows:

$$\varepsilon_{\alpha} = [\mathrm{M}(A-4, Z-2) + m({}^{4}_{2}He_{2}) - M(A, Z)]\ C^{2} \qquad ...(2\text{-}6)$$

As for the separation energy, it is defined as the energy required to supply the nucleus in order to separate a particle from it. The neutron separation energy is symbolized by the symbol (ε_{n}), which is represented by the following equation:

$$\varepsilon_{n} = [m_{n} + \mathrm{M}(A-1, Z) - M(A, Z)]\ C^{2} \qquad ...\ (2\text{-}7)$$

But if the reverse process is performed, i.e. the fusion of the nucleus (A-1,Z) and the neutron (n) to form the nucleus (A,Z), an energy will be released exactly equal to ( ), which is called the fusion energy.

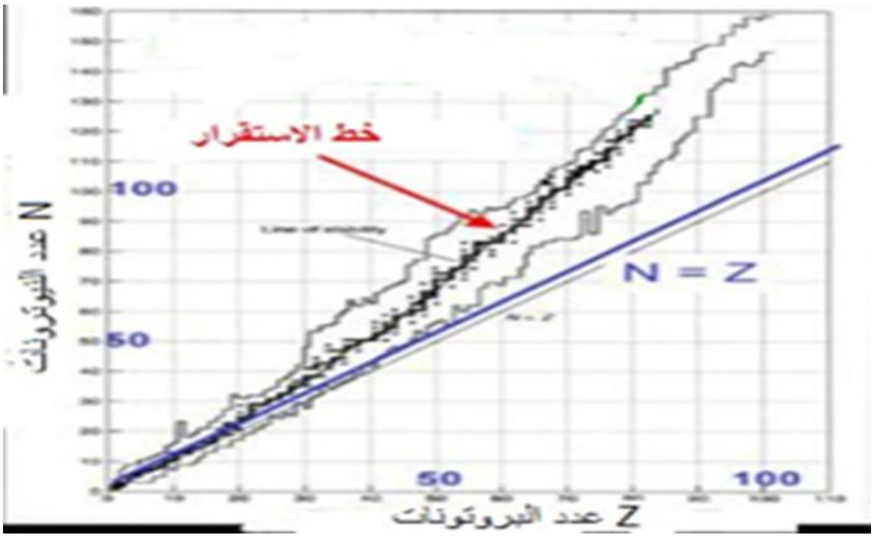
$$\varepsilon_p = [m_p + \mathrm{M}(A-1, Z-1) - M(A,Z)]\ C^2 \quad \text{... (2-8)}$$

(2-3) **Stability of the Nucleus**

There are about 2500 known nuclei in nature, of which 300 are stable and the rest are unstable (radioactive). The stability of the nucleus means that the nucleus does not emit any kind of nuclear radiation, while the unstable nuclei have the ability to emit nuclear radiation.

The factors that have an important role in knowing the stability of the nucleus are:

First: The ratio (N/Z) means the ratio of neutrons to protons, when the number of protons is equal to the number of neutrons, this ratio will be valued (1), and this means that the nuclei will be stable. As for when the number of neutrons is greater than the number of protons, the nuclei will be unstable in a direction Negative beta decay (β^- electrons), and when the number of protons is greater than the number of neutrons, the nuclei will be unstable towards the decay of positive (β^+ positrons). When a graph is drawn between the values of (N) as the y-axis and (Z) as the x-axis, it will produce a straight line that passes through the origin, as shown in Figure (2-1), where we notice from the figure that the stable nuclei are concentrated around this line (as for the unstable nuclei) In the direction of decay (β^+) they will concentrate below this line and the unstable nuclei in the direction of decay ($N < Z$) will concentrate above the line (β^-). Figure (2-1) shows that the stability line in the light nuclei coincides with the line ($N > Z$), Heavy nuclei tend to stabilize by increasing the number of neutrons to overcome the repulsive Coulomb energy of the protons.



Figure(2-1): The relationship between the number of protons and the number of neutrons to determine the stability of the nucleus.

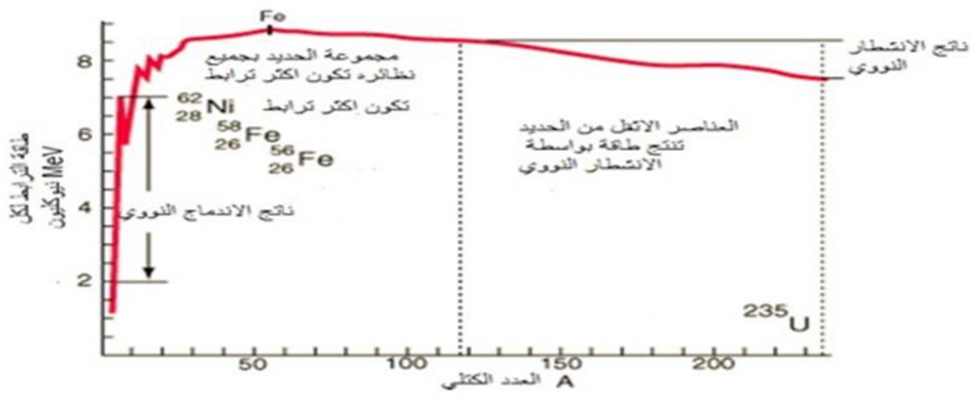
Second: The odd and even formula for the numbers of protons and neutrons have an important role in the stability of the nucleus, and Table (2-1) shows this.

Table(2.1): The stability of the nucleus as a function of the odd and even formulas for the numbers of protons and neutrons.

Z	N	الإستقرارية	عدد النوى المستقرة
فردي	فردي		4
فردي	زوجي	أقل استقرارا	50
زوجي	فردي	↓	57
زوجي	زوجي	أكثر إستقرارا	168

We notice from Table (2-1) that nuclei containing an odd number of protons and neutrons are less stable, that is, they are unsaturated nuclei, and nuclei with even numbers of neutrons and protons are more stable, that is, saturated nuclei, while nuclei with an odd number of protons Protons and an even number of neutrons are less stable than nuclei that contain an even number of protons and an odd number of neutrons.

Third: Binding energy per nucleon: The value of the binding energy per nucleon is considered a measure of the cohesion and stability of the nucleus. The higher this value, the coherent and stable the nucleus, and the lower this value, this means that the nucleus is more disintegrating and unstable. Figure (2-2) shows the binding energy for each nucleon. A nucleon as a function of mass number (A). We note from the figure that almost all nuclei possess a binding energy for each nucleon in the range (6-9) million electron-volts, and its value increases with the increase in the mass number in the light nuclei until the maximum value is reached in the range (A = 55-60), i.e. in the region of (Iron - Nickel) It is the most stable group and then its value slowly decreases when the mass number increases. Elements with low nuclear masses (light) are unstable, have very low nuclear bonding energy, and tend to fusion, As for the elements with high nuclear masses (heavy) they are unstable and have relatively low nuclear binding energy and tend to fission, while the elements with medium nuclear masses are stable and have high nuclear binding energy, so it is expected that the nuclei of heavy elements tend to fission to form nuclei of medium mass, while it is expected The nuclei of light elements fuse to form intermediate nuclei.



Figure(2-2): Binding energy per nucleon as a function of mass number.

(2-4) Nuclear Force

There are several basic forces known in nature, such as the forces of attraction, electromagnetic forces, and the weak forces (beta decay) that were known until very recently, in addition to these forces, a very important force emerged, which is the nuclear force responsible for linking the components of the nucleus with each other. Several attempts have been made to understand these forces, after the emergence of the hypothesis that the nucleus contains nucleons (protons and neutrons). The question at the time was, how are the protons and neutrons interconnected with each other inside the nucleus? In order to answer this question, we must first address the properties of nuclear power and the theories related to it, which can be summarized as follows:

The nuclear force is a strong attractive force between the nucleons inside the nucleus (proton - proton), (neutron - neutron) and (proton - neutron). This force is what maintains the stability of the nucleus. In heavy nuclei, for example, the nuclear force between nucleons is greater than the columbic repulsion force between protons, otherwise there would be no heavy nuclei.

The nuclear force has a short range within ($10^{-15}m$) and vanishes (up to zero) at a distance ($10^{-10}m$) i.e. within the size of the atom and does not follow the inverse square law. one as one particle. This force tries to make the nucleus spherical and make the nucleons in the form of pairs.

The nuclear force is saturated, meaning that each nucleon inside the nucleus can deal (interact) with force with a limited number of nucleons surrounding it. From Figure (2-2), we note that the rate of nuclear bonding energy for each nucleon is approximately equal, and this means that the nuclear force is saturated.

The nuclear force does not depend on the charge and type of the nucleon, meaning that the nuclear forces that bind two protons(P-P) or two neutrons (n-n) or a neutron and a proton (P-n) are equal.

In 1935, the scientist Hideki Yakawa hypothesized the first important theory to explain the nuclear force to explain how the nucleus is held together through an effort that combines the components of atomic nuclei of protons and neutrons. This attractive potential explains the lack of dissociation of the nucleus under the influence of the repulsion of the positively charged protons.

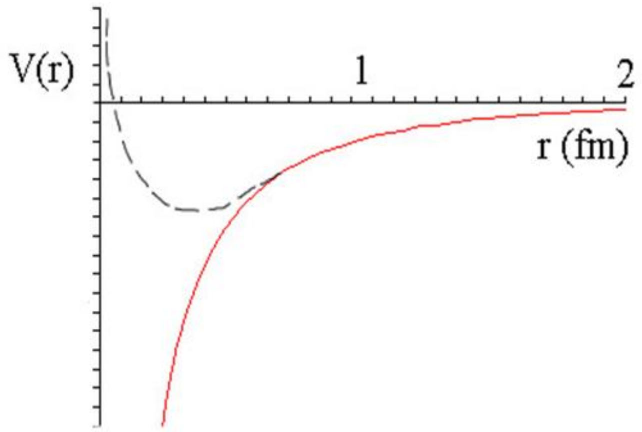
The nuclear force between nucleons occurs as a result of the exchange of particles between them, which are called pi mesons. These particles have a very short lifespan so that they cannot be detected when conducting practical experiments, so they were theoretically imposed as imaginary particles as well as being heavy particles (their mass is greater than the mass of the electron by 270 times It was referred to by the scientist Yukawa (Yakawa) in the year 5193 AD, and it was experimentally proven in the year 1937 AD, and he was awarded the Nobel Prize in 1949 AD.

To express the Yukawa voltage V(r), it can be represented by the following equation:

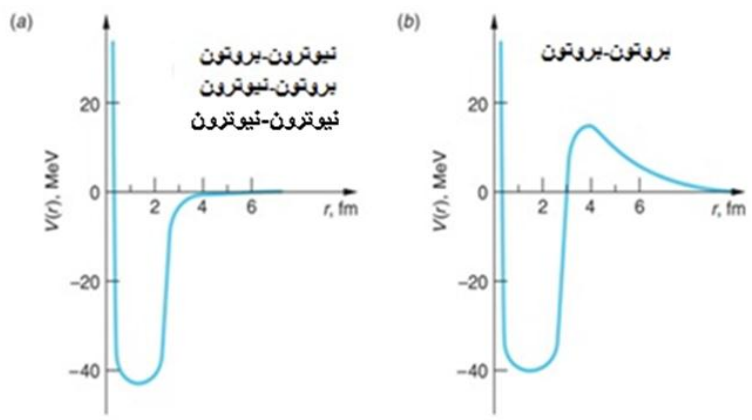
$$V(r) = -g^2 \frac{e^{-kmr}}{r} \quad ...(2\text{-}9)$$

g:constant , **m:**particles mass , **k:** Its unit constant is the inverse of distance and mass.

Figure (2-3): represents the Yukawa potential (Equation (2-9)).



Figure(2-3): Yukawa potential.



Figure(2-4): Nuclear potential between nucleons.

We notice from Figure (2-4) that the potential between two protons contains a repulsive Coulomb potential, which represents the positive part of the figure. As for the potential between a proton-neutron and a neutron-proton and a neutron-neutron, there is no positive potential because the neutron is neutral.

There are three types of pions: a zero-pion (π^0) produced when a proton turns into a proton or a neutron into a neutron, a positive-pion (π^+) when a proton turns into a neutron, and a negative ion (π^-) when A neutron is converted into a proton according to the following equations:

$$P \xrightarrow{yields} P + \pi^o$$

$$n \xrightarrow{yields} n + \pi^o$$

$$P \xrightarrow{yields} n + \pi^+$$

$$n \xrightarrow{yields} P + \pi^-$$

The emission of pi mesons leads to a loss of part of the energy of protons and neutrons, amounting to ($\Delta E = m_\pi c^2$). From the uncertainty principle (Δt.ΔE≈$\hbar$), it is possible to calculate the distance (R) that the pion travels during its transition, i.e.:

$$\therefore \Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{m_\pi C^2}$$

$$R = C.\Delta t = C.\frac{\hbar}{m_\pi C^2}$$

$$R = \frac{\hbar}{m_\pi C} = 1.4 \times 10^{-15} m = 1.4\ f \qquad ...(2\text{-}10)$$

Whereas:

$$m_\pi = 270\, m_e = 270 \times 0.00055 = 138 \quad MeV$$

We note from equation (2-10) that the value of the distance traveled by the pion is within the diameter of the nucleus, and this proves that the nuclear force is within the ionic (short-range) range.

Table (2-2) shows the four forces and their properties.

Table(2-2): The four forces, their strength, their range, and the interchangeable particles

الجسيمات المتبادلة			مداها	قوتها	القوة
البرم	الشحنة	الجسيم والكتلة السكونية			
0	0 , ±1	بايون (Pion $\pi^{\pm}$) 139.57,134.98 (MeV/C^2)	قصيرة جدا	1	النووية
1	0	فوتون Photon (0)	طويلة وتتناسب مع $1/r^2$	1/137	الكهرو مغناطيسية
1	(±e,0)	$W^{\pm}$ 81GeV/C^2) , Z^0 (91 eV/C^2)	قصيرة (0.001 (fm	10^{-9}	الضعيفة
1	0	Graviton كرافيتون (0)	طويلة وتتناسب مع $1/r^2$	10^{-38}	الجذب

(2-5) Nuclear Models

Several nuclear models have appeared to explain the structure of the nucleus, including:

(2-5-1) Liquid Drop Model

The liquid drop model is the first model that appeared to explain the structure of the nucleus. It resembles the state of atoms in a solid and molecules in a liquid. In the solid state, the atoms vibrate around fixed points in a crystalline arrangement, while the molecules in liquids move almost freely while maintaining a constant distance between them, The nuclei of atoms resemble a liquid drop in their behavior, so this model was called the liquid-drop model, meaning that the components of the nucleus move randomly within the nucleus.Von Weizsäcker in 1935 and continued to make adjustments to the constants of the equation for subsequent years.

The basic assumptions of the nucleus as a drop of liquid are that it be homogeneous in charge and not retractable, and that the nuclear force does not depend on the charge and that it is a saturated force, so nucleons interact only with their neighbors. This model is considered one of the most comprehensive, applied models and a special explanation of the phenomenon of nuclear fission (the fission of the drop when exposed to any disturbance into two equal masses) and the calculation of the mass of the nucleus and the estimation of the radius, but it failed to explain the extrapolation of nuclei that have magic numbers of protons and neutrons.

This model relied on the following energies to calculate the total bonding energy, which are as follows:

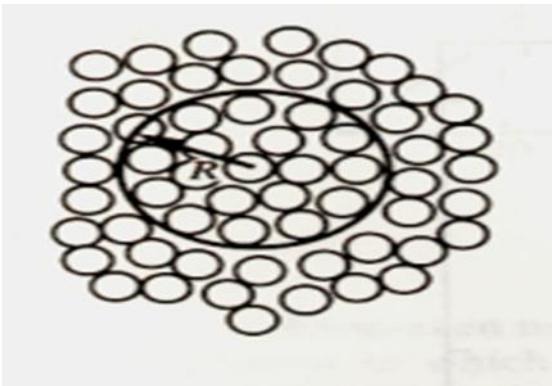
Volume Energy: We assume that the binding energy between two adjacent nuclei is (U). Since it is shared between the two nuclei, so the energy of each of them is equal to (1/2U) and when a number of drops are combined in a compact manner and each drop is surrounded by (10) drops, for example, then each drop will have an energy of (U5), and if we assume that the number of all The nuclei (A) are inside the nucleus and their volumetric energy is directly proportional to (A), so:

$$E_V = 5AU = a_v A$$

Surface Energy: There are some nuclei on the surface of the nucleus and they are surrounded by less than 10 nuclei, for example. The number of surface nuclei in the nucleus depends on its surface area. If the surface area of a nucleus, its mass number (A) is:

$$A_S = 4\pi R^2 = 4\pi R^2{}_0 A^{2/3} = -a_s A^{2/3}$$

Therefore, the number of nuclei surrounded by less than (10) nuclei is proportional to ($A^{2/3}$) as shown in Figure (2-5), and this characteristic will reduce the binding energy of the nucleus by ($E_S = -a_s A^{2/3}$) this energy plays an important role in the case of light nuclei because there is a large percentage from the surface nuclei. Since the spherical surface constitutes the smallest area surrounding a specific volume, and in the absence of an external force, the nucleus will take a spherical shape.



Figure(2-5): surface energy

Coulomb's Energy: This energy plays an important role in the binding energy and stands out in the case of medium and heavy nuclei. If we have a nucleus with atomic number (Z), then this energy is equal to the work needed to collect (Z) protons within a volume equal to the volume of the nucleus. Therefore, the Coulomb energy is directly proportional to [Z(Z-1)/2] which represents the pairs of protons in the nucleus and inversely with the radius of the nucleus ($R = R_0 A^{1/3}$), and the amount of this energy is:

$$E_C = -\frac{Z(Z-1)}{2R_\circ A^{1/3}} = -a_c \frac{Z(Z-1)}{A^{1/3}}$$

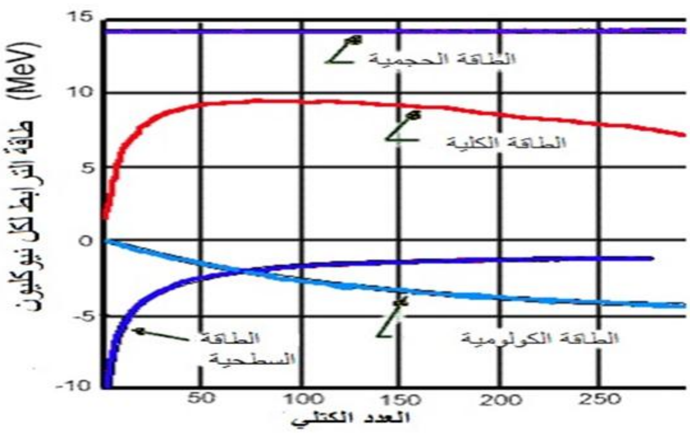
The total binding energy (E_b) of a nucleus is equal to the sum of the three energies: volume energy, surface energy, and colomb energy. It is equal to:

$$E_b = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} \qquad \textbf{...(2-11)}$$

Therefore, the binding energy per nucleon is:

$$\frac{E_b}{A} = a_v - \frac{a_s}{A^{1/3}} - a_c \frac{Z(Z-1)}{A^{4/3}} \qquad \textbf{...(2-12)}$$

Figure (2-6) shows the change of the three terms in this equation.



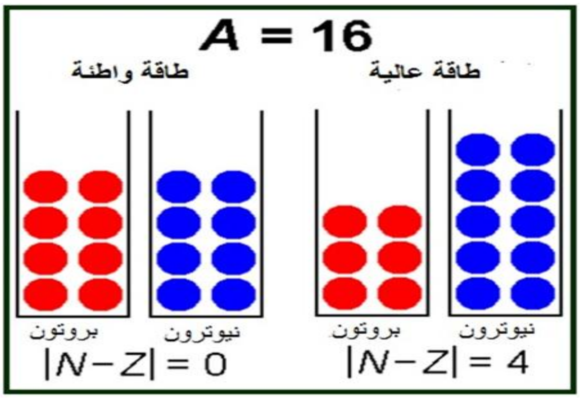
Figure(2-6): Binding energy per nucleon as a function of mass number (volume, surface, and coulomb energies).

Asymmetry Energy: It was noted that the ratio between the number of protons and neutrons in the stable light nuclei is (1), meaning that (N = Z) and it was found that this fact changes in the heavier nuclei (N ≠ Z), where their stability is relatively less due to the difference between the numbers of protons and neutrons as shown in the figure (2-7) meaning that:

$$N - Z = (A - Z) - Z = A - 2Z$$

This energy will reduce the total binding energy and equal to:

$$E_{Asy} = -a_a \frac{(A - 2Z)^2}{A}$$



Figure(2-7): Asymmetry Energy

Coupling (Pairing) Energy: The most stable and most abundant nuclei in nature are those that contain even numbers of protons and neutrons (even Z, even N), and this leads to their ability to pair, which affects the binding energy.

The coupling energy depends on a factor called the coupling factor (δ), which can be represented by the following formula:

$$E_P = \delta_P = a_P A^{-1/2}$$

Whereas:

$$\delta_P = +a_P A^{-1/2} \qquad for\ \ even\ Z - even\ N$$

$$\delta_P = 0 \qquad for\ \ odd\ Z - even\ N \qquad and \qquad even\ Z - odd\ N$$

$$\delta_P = -a_P A^{-1/2} \qquad for\ \ odd\ Z - odd\ N$$

Therefore, the binding energy can be written as:

$$B.E = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + E_P \qquad ...(2\text{-}13)$$

Therefore, the semi-empirical formula for calculating the mass of the nucleus, which is called the Bethe-Weizaecker formula, can be written as follows:

$$M(Z,A) = Zm_p + Nm_n - B.E/c^2$$

$$M(Z,A) = Zm_p + Nm_n - (a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + E_P)/c^2$$

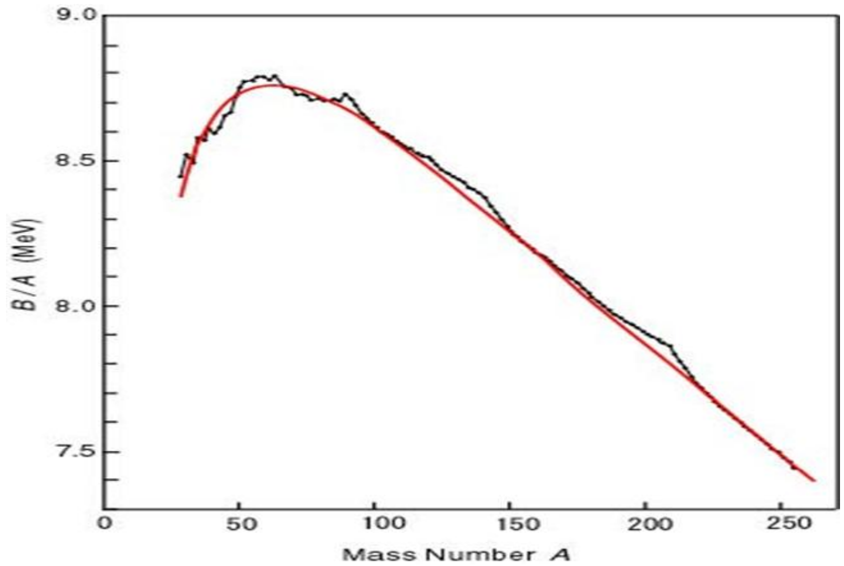
...(2-14)

The constants of equations (2-13) and (2-14) are:

$a_v = 15.85\ MeV\ \ , a_s = 18.34\ \ MeV\ ,$

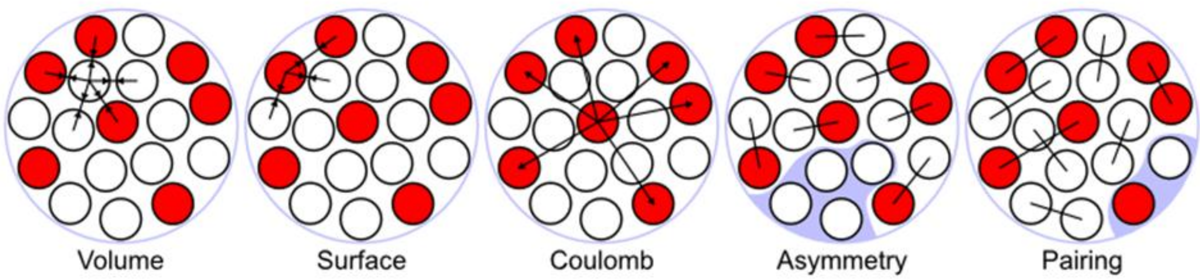
$a_c = 0.714\ MeV\ ,\ \ a_a = 23.21\ MeV , a_p\ \ \ = 12\ \ MeV$

These constants showed great agreement with the experimental values, as the results of the bonding energy per nucleon that were calculated from equation (2-13) (the straight curve) were compared with the experimental results (the zigzag curve) as shown in Figure (2-8).



Figure(2-8): Theoretical and experimental bonding energy per nucleon as a function of mass number.

Figure (2-9): shows the energies on which the total binding energy depends and shown in equation (2-14).



Figure(2-9): The total binding energy according to the liquid drop model.

(2-5-2) Shell Model

The main characteristic of the liquid-drop model is that the components of the nucleus interact only with its neighbors, and there are experimental phenomena that support this fact. However, there are other phenomena that indicate the opposite, such as the influence of nuclei on the field of the nucleus as a whole instead of the fields of individual nuclei, and this characteristic leads to nuclear states that are somewhat similar to the atomic states. Because the liquid drop model failed to explain the stability of the nucleus and the nature of the nuclear force, the shell model appeared to explain some phenomena that the liquid drop model could not explain, which depends on the Pauli exclusion principle to describe the structure of the nucleus using energy levels.

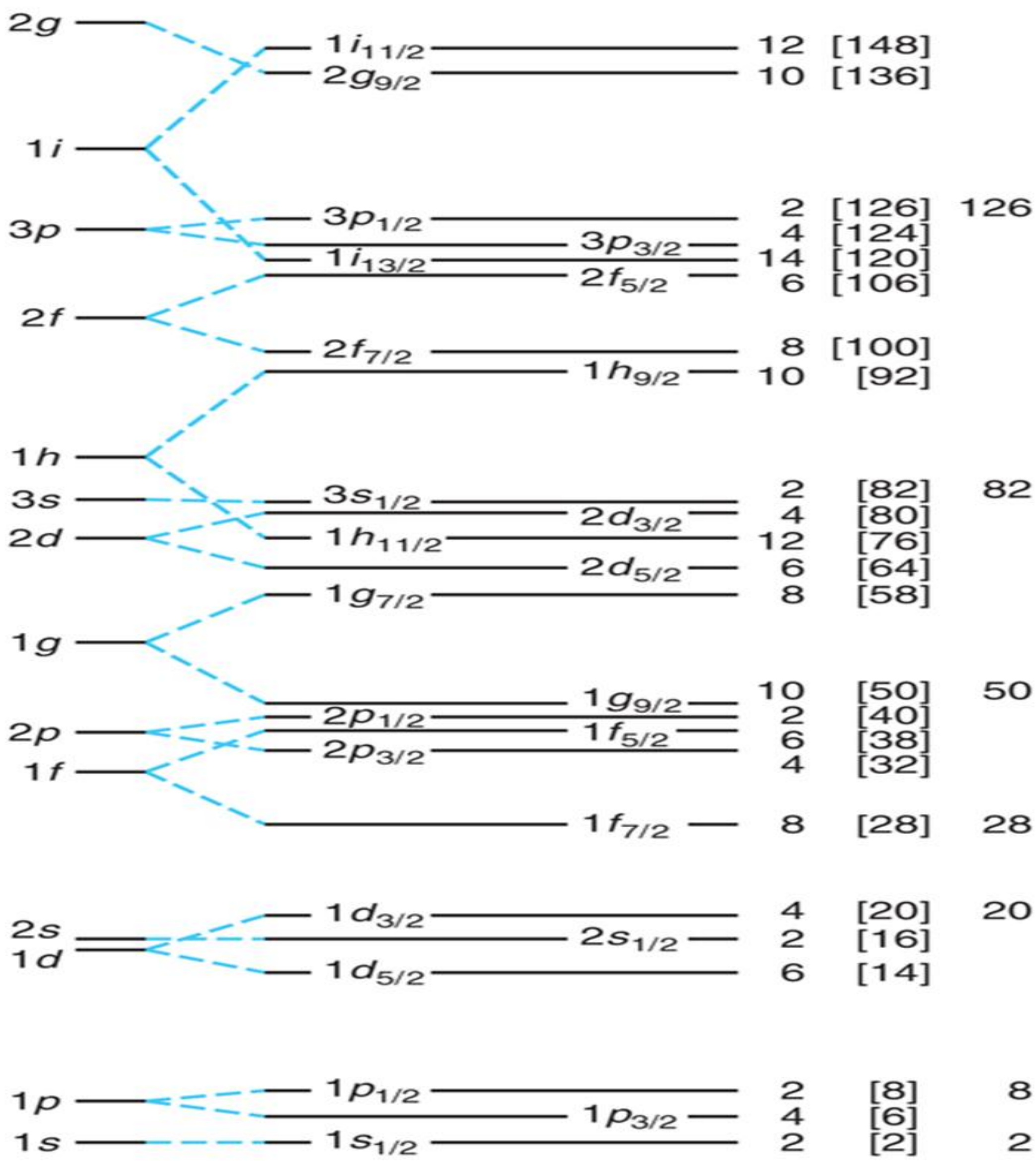
The first cortical model was proposed in 1932 by the scientist dmitry Ivanenko, and this model was developed in 1949 by the scientists Eugene Paul Wigner, Maria Kupertmaier and J. Hans Jensen (Eugene Paul Wigner, Maria Goeppert-Mayer and J. Hans D. Jensen). They were awarded the Nobel Prize in 1963 AD for their contribution to this achievement.

The electrons in an atom can be imaging as occupying shells that are determined by certain quantitative numbers, while the degree of occupancy of the outer shell determines the chemical properties of the atom. For example, the outer shells with atomic numbers 2, 8, 18, 32, 50 and 72 are saturated, so the electronic structures of these atoms are more stable, and this explains the inactivity of rare gases. The same characteristic can be observed in the case of nuclei with numbers 2, 8, 20, 28, 50, 82 and 126 protons or neutrons, as we find that these nuclei are more stable and more abundant in nature.

There are other observations that indicate the importance of the numbers 2, 8, 20, 28, 50, 82 and 126 in the nuclear structure, so these numbers are called the magic number, and from these observations is the nuclear quadruple moment, which represents the distance distribution The nuclear charge is about the spherical distribution, the spherical nucleus does not have a quadruple moment, while the oval-shaped nucleus has a positive moment and the pear-shaped nucleus has a negative moment.

The distribution of protons and neutrons is similar to the distribution of electrons in terms of levels, meaning that there is a main level that takes the quantum number (n) and the orbital quantum number (ℓ) of the particle, and the total angular momentum (J). As in the case of electronic levels, the interaction of spinning with the orbital leads to the splitting of each state with a value (J) into (2J+1) secondary state, and the distances between the secondary levels are small relative to the distances between the basic levels, so that the successive crusts and the number of states can be distinguished. The shells are full when there are 2, 8, 20, 28, 50, 82, and 126 neutrons or a proton in the nucleus.

Figure (2-10) shows how protons and neutrons are distributed in energy levels.



Figure(2-10):The sequence of energy levels according to the shell model

There are some facts related to magic numbers, including:

1- The nuclei with magic numbers have isotopes of protons (Isotopes) and isotopes of neutrons (Isotones) compared to other nuclei such as zinc nuclei (^{50}Zn), which have (10) isotopes.

2- The neutron separation energy of these nuclei is very high.

3- The magic nuclei are more stable and more interconnected, which makes them more abundant in nature.

The shell model can explain several nuclear phenomena in addition to magic numbers:

The nuclei are (even Z , even N): The levels of these nuclei are filled with even numbers of protons and neutrons in the secondary levels, and they are more stable and the total angular momentum is zero, as the perpendicular and orbital momentum of these nuclei erase each other, and there are about 160 even-even stable nuclei.

The nuclei are (odd Z , odd N): The levels of these nuclei are filled with odd numbers of protons and neutrons in the secondary levels and are less stable, and the total angular momentum (the sum of the spin and orbital momentum of these nuclei) is equal to an integer, and there are only four stable odd-odd nuclei $^{2}_{1}H_1$ و $^{6}_{3}Li_3$ و $^{10}_{5}B_5$ و $^{14}_{7}N_7$.

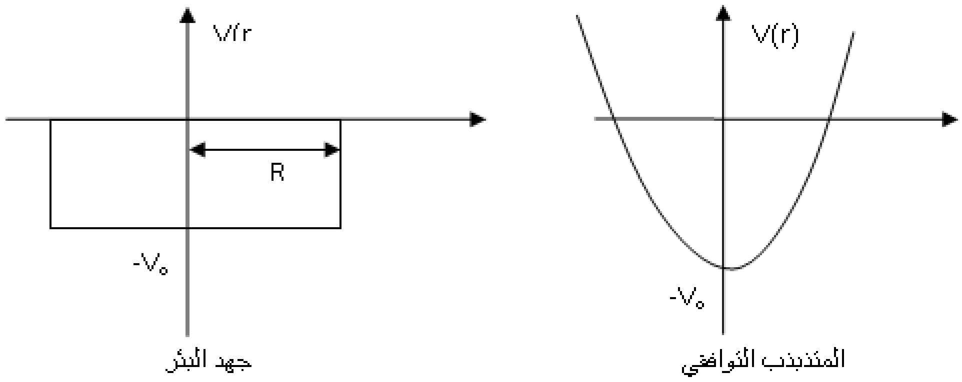
The nuclei are even-odd or the nuclei are even-odd: The levels of these nuclei contain a single particle, either a proton or a neutron in the secondary levels, and the total angular momentum (the sum of the spin and orbital momentum of these nuclei) is equal to half of the integers.

Theory of Shell Model

The one-particle shell model is based on two main assumptions:

1- Every nucleus moves freely and fluently in the force field, which is expressed as potential.
2- Applying the Pauli Selection Principle, meaning that the energy levels or shells are filled with respect to the Selection Principle.

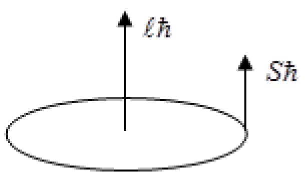
According to quantum mechanics, we will show a general model that includes the existence of shells or nuclear levels using the nucleus voltage V(r) in the form of the square well potential and the harmonic oscillator potential on the basis that the nuclei are spherical, as shown in Figure (2-11).

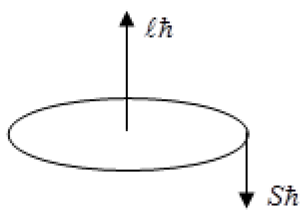


Figure(2-11): Harmonic oscillator voltage and well voltage.

In nuclear physics, the state (the plane) is known by two quantum numbers, which are the principal quantum number (n) and the orbital quantum number (ℓ), and it was found when distributing the nuclei of any stable nucleus in any of the two voltages that it cannot cover all the magic numbers, so in 1949 a number of Scientists came up with the idea of nuclear coupling between the orbital angular momentum and the perfusion angular momentum of each nucleus. This model was called the spin-orbit coupling model, meaning:

$$J = \ell \mp s \qquad , \qquad J = \ell \mp \frac{1}{2}$$





Momentums are parallel direction

The momentum is opposite in

In the case (2P), it splits into two secondary levels due to the interaction of spinning with the orbital to give the state (2P1/2) and the state (2P3/2) and these levels are filled with (2J+1) nuclei, i.e.:-

$$2P_{1/2} \rightarrow (2\times 1/2)+1=1+1=2$$
$$2P_{3/2} \rightarrow (2\times 3/2)+1=3+1=4$$

The symmetry $(-1)^{\ell}$ can be calculated from the orbital quantum number (ℓ) of the last proton or the last neutron. If the orbital quantum number is even, the symmetry will be even (positive), but if it is odd, the symmetry will be odd (negative).

The angular momentum (nuclear spin) and nuclear symmetry for any nucleus in the ground state can be estimated according to the following two assumptions:

1-In filled planes,the orbital and perpendicular angular momentum are combined in such a way that the total angular momentum is equal to zero.

2- In unfilled levels, the nucleus forms pairs of protons or neutrons, not pairs of protons and neutrons.

Based on these two assumptions, we can arrive at the following two basic rules:

First rule: The total angular momentum* for any nucleus in the ground state whose number of nucleons (even - even) is equal to zero and therefore the symmetry will be positive.

$$\sum J_n = 0 \quad , \quad \sum J_P = 0$$

Second rule:

a- The nucleus in which (odd Z , even N) is and the ground state for spinning the nucleus depends on spinning the last proton.

b- The nucleus in which (even Z, odd N) is spinning and the ground state for spinning the nucleus depends on spinning the last neutron.

c- The nucleus in which (odd Z, odd N) is calculated by calculating the ground state in relation to spinning the nucleus from the last proton and the last neutron ($J_n - J_P$, $J_n + J_P$), and the symmetry in this case is calculated from the product of the symmetry of the last proton and the symmetry of the last neutron.

The crustal model was able to deduce magic numbers and angular moments, and failed to explain the phenomena related to the electric quadruple moments, because the nucleons far from the filled crust possess a large electrical moment, which means that these nuclei are not spherical.

(2-6) solved Examples

Example(1)

Find the bonding energy of beryllium ($^{9}_{4}Be_5$) with respect to its dissolution into a neutron and two helium nuclei ($^{4}_{2}He_2$), then find the total bonding energy of a beryllium nucleus.

$$^{9}_{4}Be_5 \rightarrow {}^{1}_{0}n + 2\,{}^{4}_{2}He_2$$

$$B.E({}^{9}_{4}Be_5) = [m_n + 2M({}^{4}_{2}He_2) - M({}^{9}_{4}Be_5)]c^2$$

$$= [1.008665 + 2\times 4.002603 - 9.012187]\times 931.5 = 2MeV$$

$$B.E = [Zm_H + Nm_n - M(A,Z)]c^2$$

$$= [4\times 1.007825 + 5\times 1.008665 - 9.012182]\times 931.5$$

$$= 58MeV$$

Example(2)

Calculate the proton separation energy (ε_p) for the mirror nuclei ($^3_1H_2 - ^3_2He_1$) and interpret the results.

Sol:

$$^3_1H_2 \rightarrow ^1_1p + ^1_0n + ^1_0n$$

$$\varepsilon_p(^3_1H_2) = [m_p + 2m(^1_0n) - M(^3_1H_2)]c^2$$

$$= [1.0072766 + 2 \times 1.0086654 - 3.016049] \times 931.5$$

$$= 7.97 MeV$$

$$^3_2He_1 \rightarrow ^1_1p + ^2_1H_1$$

$$\varepsilon_p(^3_2He_1) = [m_p + M(^2_1H_1) - M(^3_2He_1)]c^2$$

$$= [1.0072766 + 2.01410177 - 3.016029319] \times 931.5$$

$$= 5 MeV$$

We note from the above results that the energy to separate a proton from the nucleus of a tritium atom (^3_1H) is greater than the energy to separate a proton from the nucleus of a helium atom(^3_2He)due to the columbic repulsion force between the protons of the helium nucleus, which leads to a decrease in the bonding energy between nucleons, which It is easy to separate the proton from it, and therefore it will need less energy, and this is what the above results proved.

Example(3)

Is the beryllium nucleus ($^8_4Be_4$) stable for alpha particle decay? Explain this.

Sol:

$$^{8}_{4}Be_4 \longrightarrow ^{4}_{2}He_2 + ^{4}_{2}He_2$$

$$B.E(^{8}_{4}Be_4) = [2M(^{4}_{2}He_2) - M(^{8}_{4}Be_4)]c^2$$

$$= [2 \times 4.002603254 - 8.022487] \times 931.5$$

$$= -16.097 MeV$$

The negative sign indicates that the beryllium nucleus is unstable to the decay of alpha particles.

Example(4)

What is the minimum energy that can be expended for a fluorine nucleus $^{19}_{9}F_{10}$) to split into a proton, a nucleus ($^{7}_{3}Li_4$) and a nucleus ($^{11}_{5}B_6$)?.

$$^{19}_{9}F_{10} \longrightarrow ^{1}_{1}p + ^{7}_{3}Li_4 + ^{11}_{5}B_6$$

$$B.E(^{19}_{9}F_{10}) = [m_p + M(^{7}_{3}Li_4) + M(^{11}_{5}B_6) - M(^{19}_{9}F_{10})]c^2$$

$$= [1.00727663 + 7.0159772 + 11.0092793 - 18.998403] \times 931.5$$

$$= 31.02668 MeV$$

The minimum energy needed by the nucleus ($^{19}_{9}F_{10}$) for the above cleavage is ($31.02668 MeV$).

Example(5)

Prove that the nuclear force does not depend on the charge by taking the two mirror nuclei ($^{3}_{1}H$, $^{3}_{2}\mathrm{He}$) .

Sol:

The bonding energy of these two nuclei is calculated as follows:

$$\mathrm{B.E}(^{3}_{1}H) = (m_H + 2 \times m_n - M(^{3}_{1}H)) \times 931.5$$

$$= (1.007825 + 2 \times 1.008665 – 3.016049) \times 931.5$$

$$= 8.5 \text{ MeV}$$

$$\text{B.E}(^3_2He) = \left(2 \times m_H + m_n - M(^3_2He)\right) \times 931.5$$

$$= \left(2 \times 1.007825 + 1.008665 - 3.016029\right) \times 931.5$$

$$= 7.72 \text{ MeV}$$

There is a difference in the bonding energy between the two nuclei resulting from the repulsion force between the two protons of the helium nucleus, so we will calculate the Coulomb energy as follows:

$$E_C = k\frac{e^2}{R} = 0.82 \text{ MeV}$$

$$\therefore \quad 7.72 + .82 = 8.5 \quad MeV$$

We note from the above equations that the bonding energy of a helium nucleus is equal to the bonding energy of a tritium nucleus, and this proves that the nuclear force does not depend on the charge.

Example(6)

Calculate the mass of deuterium ^2_1D ($^2_1$H), if the bonding energy per nucleon is1.02MeV.

Sol:

$$B.E = [m_H + Nm_N - M(A,Z) \times 931.5$$

$$= [1 \times 1.007825 + 1 \times 1.008665 – M(^2_1D)] \times 931.5$$

$$= [2.01649 – M(^2_1D)] \times 931.5$$

$$\frac{B.E}{A} = \frac{[2.01649 - M(^2_1D)]}{2} = \frac{1.02}{931.5}$$

$$0.002190016 = 2.01649 – M(^2_1D)$$

$$m(^2_1D) = 2.014299 \, amu$$

$1amu = 1.66 \times 10^{-27}$ kg

$M({}_1^2D) = 2.014299 \times 1.66 \times 10^{-27}$

$= 3.344 \times 10^{-27}$ kg

Example(7)

Calculate the coupling energy using the liquid drop model for the following nuclei: ${}_6^{12}C$, ${}_7^{13}N$, ${}_7^{14}N$

Sol:

As for the carbon-12 nucleus, it has the number of protons (6) and the number of neutrons (6), meaning that it is an even-even nucleus, so the pairing energy is:

$$E_p = +a_p A^{-1/2} = 12 \times (12)^{-1/2} = 3.46 \text{ MeV}$$

As for the nitrogen-13 nucleus, it has the number of protons (7) and the number of neutrons (6), meaning that it is an odd-even nucleus, so the pairing energy is:

$$E_p = 0$$

As for the nitrogen-14 nucleus, it has the number of protons (7) and the number of neutrons (7), meaning that it is an odd-odd nucleus, so the pairing energy is:

$$E_p = -a_p A^{-1/2} = 12 \times (14)^{-1/2} = -3.207 \text{ MeV}$$

Example(8)

Calculate the volumetric energy and surface energy according to the liquid-drop model of the boron-10 nucleus.

Sol:

The volumetric energy is calculated from the following equation:

$$E_v = a_v A = 15.85 \times 10 = 158.5 \text{ MeV}$$

The surface energy is calculated from the following equation:

$$E_s = -a_S A^{\frac{2}{3}} = -18.34 \times 10^{2/3} = -85.1267 \text{ MeV}$$

Why is the surface energy less than the volumetric energy? Leave the answer to the student.

Example(9)

Find the nuclear spin and symmetry of the ground state of the calcium nucleus, which contains (Z=20) protons and (N=23) neutrons, according to the shell model?

Sol:

The levels can be filled with protons and neutrons according to the shell model, as follows:

First: Filling the first level (S-state) ($\ell = 0$), the total angular momentum is calculated from the sum of the spin of the proton (S=1/2) and the orbital angular momentum ($\ell = 0$), i.e. ($j=\ell+s$), j=1 /2)), so the number of protons will be (2j+1=2(1/2)+1=2), and the plane will be filled with two protons. It will be the same for neutrons.

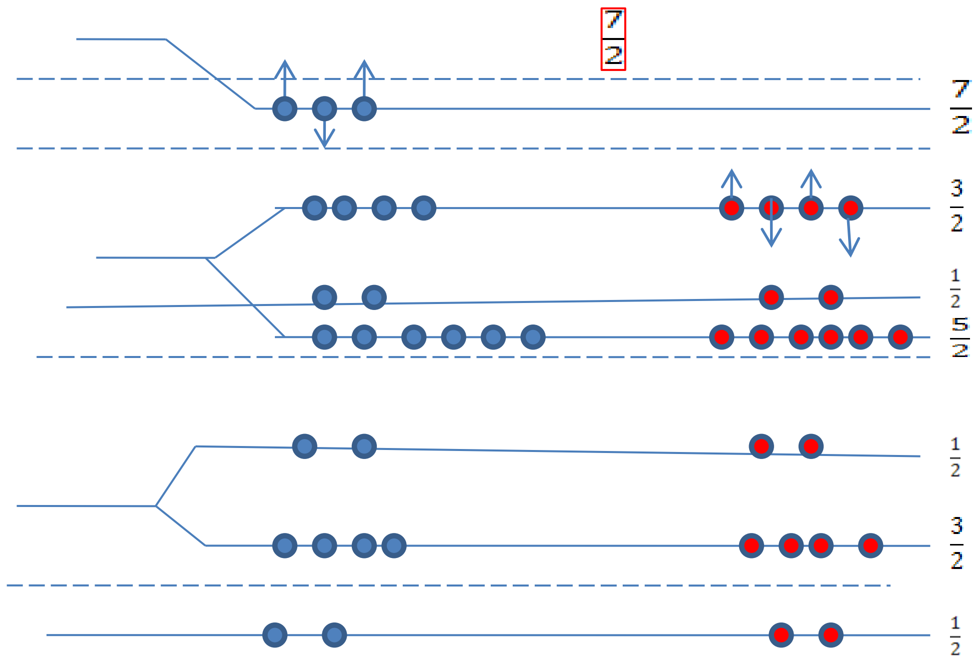
Second: Filling the second plane (state-p) ($\ell = 1$), calculating the total angular momentum from the sum of the spin of the proton (S=1/2) and the orbital angular momentum ($\ell = 1$) i.e. $j= \ell \pm s$) j=1 /2, 3/2)), and here we will have two levels, the first will be filled with two protons 2j+1=2(1/2)+1=2)), and the second will be filled with four protons 2j+1=2(3/2) +1=4)). It will be the same for neutrons. And so for the rest of the levels, where the distribution continues until reaching the level (f).

The ground state is calculated from the last single proton or neutron in the last plane.

The figure below shows the distribution of protons and neutrons for a calcium-43 nucleus. From the figure, we notice that the ground state of the nucleus is ($1\ f_{7/2}$), meaning that the nuclear spin is equal to (7/2), and the symmetry will be:

$f - state \Rightarrow\ l = 3$ ، $\therefore \pi = (-1)^{\ell} = (-1)^3 = -1$ (odd).

So, the ground state of the calcium-43 nucleus is: $((7/2)^{-}$.



$^{43}_{20}Ca$

Example(10)

Find the nuclear spin and symmetry according to the shell model for the following nuclei:

$^{7}_{3}$Li . $^{11}_{5}$B, $^{15}_{6}$C, $^{17}_{9}$F, $^{31}_{15}$P, $^{141}_{59}$Pr, $^{12}_{5}$B, $^{16}_{8}$O , $^{17}_{8}$O , $^{14}_{7}$N

Note: The nuclear spin of the even numbers of nucleons is equal to zero according to the shell model.

Sol:

1.Calculate the nuclear spin of the nucleus of lithium(${}^{7}_{3}Li$)Li. by another proton because the number of protons is odd in the plane and as follows:

$$Z = 3 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{1}\left(\mathrm{J} = \frac{3}{2}\right), N = 4 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{2}(\mathrm{J} = 0)$$

$$\pi = (-1)^{\ell} = (-1)^{1} = -1$$

$$\mathrm{J}^{\pi} = 3/2^{-}$$

2.The nuclear spin of the boron nucleus ${}^{11}_{5}B$ is calculated by the last proton because the number of protons is odd in the plane, as follows:

$$\mathrm{Z} = 5 \rightarrow 1\mathrm{S}_{\frac{1}{2}}{}^{2}, 1\mathrm{P}_{\frac{3}{2}}{}^{3}\left(\mathrm{J} = \frac{3}{2}\right), \mathrm{N} = 6 \rightarrow 1\mathrm{S}_{\frac{1}{2}}{}^{2}, 1\mathrm{P}_{\frac{3}{2}}{}^{4}(\mathrm{J} = 0)$$

$$\pi = (-1)^{\ell} = (-1)^{1} = -1$$

$$\mathrm{J}^{\pi} = 3/2^{-}$$

3. The nuclear spin of the carbon nucleus ${}^{15}_{6}C$ is calculated by the last neutron because the number of neutrons is odd in the plane, as follows:

$$\mathrm{Z} = 6 \rightarrow 1\mathrm{S}_{\frac{1}{2}}{}^{2}, 1\mathrm{P}_{\frac{3}{2}}{}^{4}(J = 0) \quad ,$$

$$\mathrm{N} = 9 \rightarrow 1\mathrm{S}_{\frac{1}{2}}{}^{2}, 1\mathrm{P}_{\frac{3}{2}}{}^{4}, 1\mathrm{P}_{\frac{1}{2}}{}^{2}, 1\mathrm{d}_{\frac{5}{2}}{}^{1}(\mathrm{J} = \frac{5}{2})$$

$$\pi = (-1)^{\ell} = (-1)^{2} = +1$$

$$\mathrm{J}^{\pi} = 5/2^{+}$$

4. The nuclear spin of the fluorine nucleus ${}^{17}_{9}F$is calculated by the last proton because the number of protons is odd in the plane, as follows:

$$\mathrm{Z} = 9 \rightarrow 1\mathrm{S}_{\frac{1}{2}}{}^{2}, 1\mathrm{P}_{\frac{3}{2}}{}^{4}, 1\mathrm{P}_{\frac{1}{2}}{}^{2}, 1\mathrm{d}_{\frac{5}{2}}{}^{1}\left(\mathrm{J} = \frac{5}{2}\right) \quad ,$$

$$\mathrm{N} = 8 \rightarrow 1\mathrm{S}_{\frac{1}{2}}{}^{2}, 1\mathrm{P}_{\frac{3}{2}}{}^{4}, 1\mathrm{P}_{\frac{1}{2}}{}^{2}, (\mathrm{J} = 0)$$

$\pi = (-1)^{\ell} = (-1)^{2} = +1$

$J^{\pi} = 5/2^{+}$

5. Calculate the nuclear spin of the nucleus of phosphorous ${}^{31}_{15}P$by the last proton because the number of protons is odd in the plane, as follows:

$$Z = 15 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{4}, 1P_{\frac{1}{2}}{}^{2}, 1d_{\frac{5}{2}}{}^{6}, 2S_{\frac{1}{2}}{}^{1}, \left(J = \frac{1}{2}\right)$$

$$N = 16 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{4}, 1P_{\frac{1}{2}}{}^{2}, 1d_{\frac{5}{2}}{}^{6}, 2S_{\frac{1}{2}}{}^{2} (J = 0)$$

$\pi = (-1)^{\ell} = (-1)^{0} = +1$

$J^{\pi} = 1/2^{+}$

6. Calculate the nuclear spin of the praseodymium nucleus ${}^{141}_{59}\mathrm{Pr}$by the last proton because the number of protons is odd in the plane, as follows:

$$Z = 59 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{4}, 1P_{\frac{1}{2}}{}^{2}, 1d_{\frac{5}{2}}{}^{6}, 2S_{\frac{1}{2}}{}^{2}, 1d_{\frac{3}{2}}{}^{4} ,$$

$$1f_{\frac{7}{2}}{}^{8}, 1f_{\frac{5}{2}}{}^{6}, 2P_{\frac{3}{2}}{}^{4}, 2P_{\frac{1}{2}}{}^{2}, 1g_{\frac{9}{2}}{}^{10}, 1g_{\frac{7}{2}}{}^{8}\ 2d_{\frac{5}{2}}{}^{1} \left(J = \frac{5}{2}\right)$$

$$N = 82 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{4}, 1P_{\frac{1}{2}}{}^{2}, 1d_{\frac{5}{2}}{}^{6}, 2S_{\frac{1}{2}}{}^{2}, ----, (J = 0)$$

$\pi = (-1)^{\ell} = (-1)^{2} = +1$

$J^{\pi} = 5/2^{+}$

7. The nuclear spin of the boron nucleus ${}^{12}_{5}\mathrm{B}$ is calculated by the last proton because the number of protons is odd in the plane, as well as by the last neutron because the number of neutrons is also odd in the plane, as follows:

$$Z = 5 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{3} \left(J = \frac{3}{2}\right) ,$$

$N = 7 \rightarrow 1S_{\frac{1}{2}}{}^{2}, 1P_{\frac{3}{2}}{}^{4}$ و $1P_{\frac{1}{2}}{}^{1}\left(J = \frac{1}{2}\right),$

$$\therefore \quad J = \frac{3}{2} \mp \frac{1}{2} = 2{,}1$$

$$\pi_p = (-1)^{\ell} = (-1)^1 = -1, \qquad \pi_n = (-1)^l \quad = -1$$

$$\because \pi = \pi_p \times \pi_n = +1$$
$$\therefore \; J^{\pi} \quad = 2^+, 1^+$$

8. Oxygen-16

$$Z = 8 \;, N = 8$$

Since the number of protons and neutrons is even, the nuclear spin will be zero and the symmetry is even (positive).

9. Oxygen-17

$$Z = 8 \;, N = 9$$

In this case, the nuclear spin and symmetry will be calculated from the last neutron and these neutrons will be filled as follows:

$$(1S_{\frac{1}{2}})^2 \quad ,(1P_{\frac{3}{2}})^4 \quad ,(1P_{\frac{1}{2}})^2 \quad ,(1d_{\frac{5}{2}})^1$$

The last neutron will be in the plane ($1d_{5/2}$), so the nuclear spin is equal to (5/2), and the symmetry is calculated as follows:

$$d - \text{state} \;\Rightarrow \ell = 2$$

So the symmetry

$$(-1)^{\ell} = (-1)^2 \;=\; +1$$

So the spin and symmetry of the nucleus $^{17}_{8}O$ will be $(5/2)^+$

10. Nitrogen -14

$Z = 7 \quad , \quad N = 7$

Since the number of neutrons and the number of protons is odd, so for protons it will be.

$\therefore J_p = \frac{1}{2} \quad , \quad \ell = 1$

The symmetry will be (odd).

$\pi_P = (-1)^{\ell} = (-1)^1 = -1$

As for the neutrons:

$N = 7 \Rightarrow (1S_{\frac{1}{2}})^2 \quad , (1P_{\frac{3}{2}})^4 \quad , (1P_{\frac{1}{2}})^1$

$\therefore J_n = \frac{1}{2} \quad , \quad \ell = 1$

The symmetry will be:

$\pi_n = (-1)^{\ell} = (-1)^1 = -1$ (odd)

So the nuclear spin would be J_P-J_n and J_P+ J_n So the nuclear spin would be:

$J = 0 \; , \; 1$

The symmetry will be the product of the symmetry of the last proton and the last neutron as follows:

$= \pi_P \times \pi_n = (-1)(-1)$
$= +1$ (even)

So, the ground state of the nucleus is:

$1^+ \text{or } 0^+$

Chapter two questions and problems

Q1/ Define binding energy and what is the relationship between it and the stability of the nucleus?

Q2/ What is the difference between the energy of emitting a neutron and the energy of emitting a proton from a nucleus, which is greater and why?

Q3/ What is the idea of the liquid drop model of the nucleus?

Q4/ What are the two hypotheses of the shell model?

Q5/ Write four nuclei with a magic number of nucleons.

Q6/ Calculate the total binding energy for the following nuclei ($^{16}_{8}O_{8}$),($^{4}_{2}He_{2}$) and ($^{12}_{6}C_{6}$).

Q7/ Find the proton separation energy and the neutron separation energy of the two nuclei ($^{4}_{2}He_{2}$) and ($^{2}_{1}H_{1}$), and what can you conclude from the results that you will get?

Q8/ Calculate the binding energy for each nucleon and compare the results for the following nuclei $^{208}_{82}Pb_{126}$, $^{62}_{28}Ni_{34}$, $^{6}_{3}Li_{3}$.

Q9/ Which is more stable, the magnesium nucleus ($^{24}_{12}Mg_{12}$) or the aluminum nucleus ($^{27}_{13}Al_{14}$)?.

Q10/ How much energy is required to supply an oxygen nucleus ($^{16}_{8}O_{8}$) in order for it to split into four equal nuclei?

Q11/ How much energy is needed to split the lithium nucleus ($^{6}_{3}Li_{3}$) into three equal nuclei?

Q12/ Calculate the energy of separating a proton and a neutron from a nucleus ($^{3}_{2}He_{1}$), and is there a difference between them and why?

Q13/ Calculate the energy of separating the nucleus of a helium atom $^{4}_{2}He_{2}$ from the nucleus ($^{13}_{6}C_{7}$).

Q14/ Is there a difference between the binding energy of another proton and another neutron in a helium-4 nucleus, prove that and why?

Q15/ Calculate the mass of the nucleus ($^{20}_{10}Ne_{10}$) according to the liquid drop model.

Q16/ Distribute the nucleons according to the shell model for the following nuclei ($^{36}_{18}Ar_{18}$), ($^{35}_{17}Cl_{18}$) and ($^{32}_{16}S_{16}$).

Q17/ Calculate the difference in the total binding energy of the iron nucleus ($^{56}_{26}Fe$) when it is calculated using the liquid drop model and calculated by decreasing mass.

Q18/ Calculate the asymmetry energy of the tellurium nucleus -120 ($^{120}_{52}$Te).

Q19/ Which is greater, the surface energy or the volumetric energy of a uranium nucleus-238 ($^{238}_{92}U$)?

Q20/ Find the spin and symmetry of the nucleus of barium-137($^{137}_{56}Ba$) using the shell model.